PROBLEM ON 2009 OCTOBER 23

MVHS NUMBER THEORY GROUP

In Western music (i.e. almost all the music we listen to or play today) the most important intervals are the unison, octave and the perfect fifth. These intervals date back to ancient Greece when mathematicians (such as Pythagoras) and musicians were experimenting with different intervals. They found *frequency ratios* [1] to be most pleasing when their values were positive integers. It is no surprise then that we have the following frequency ratios for these important intervals.

unison	$1:1$
octave	$1:2$
perfect fifth	$2:3$

Once the Greeks had these ratios, they proceeded to try and construct a musical system that incorporated what they had found. However, it wasn't so easy. The Greeks had decided from the start that their musical scale would be *equal tempered* [2], unbeknown to the difficulties it would cause.

Your job is to explain why there does NOT exist an equal tempered scale which incorporates both the octave and the perfect fifth. As a hint, try to think about what this would mean in terms of the ratios for the octave and the perfect fifth. Having an equal tempered scale incorporating both intervals would mean finding integers n and m such that

$$\left(\frac{2}{1}\right)^n = \left(\frac{3}{2}\right)^m$$

Wednesday's notes tell exactly why no such integers exist. Experiment with some values for n and m trying to make the above equality approximate. Can you deduce why today's system of music is composed of the twelve equal tempered semitones with frequency ratio $1 : 2^{1/12}$? This problem is worth **1 Point**.

[1] To get an idea about what a frequency ratio is, think of a guitar. If we pluck a string we get the unison sound by splitting the string up into 1 equal part this is the ratio $1 : 1$. If we split the string into 2 equal pieces or halves, and hold down in the middle we get the octave with ratio $1 : 2$. Similarly, if we split the string into 3 equal pieces or thirds, and hold down between the second and third piece we get the ratio $2 : 3$ which is the perfect fifth.

[2] Equal tempered means that the *distance* between every consecutive note is the same.